

Some Characteristic Parameters of Proton from the Bag Model

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We treat the mass of a proton as the total static energy which can be separated into two parts that come from the contribution of quarks and gluons respectively. We adopt the essential of the bag model of hadron to discuss the structure of a proton and find that the calculated temperature, proton radius, the bag constant are compare well with QCD results if a proton is a thermal equilibrium system of quarks and gluons.

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I. INTRODUCTION

Exploring proton structure [1–3] is still one of most important subjects for more profound enhancement of human knowledge on strong interactions. It is also very helpful for people to search for a new matter state—quark gluon plasma (QGP) which is the deconfined state of strongly interacting matter. Most theoretical investigations focus on Quantum Chromodynamics (QCD) [4]. However, there are a few phenomenological models about nucleon structure and interactions. The classical string model describes mesons as string segments executing longitudinal expansion and contraction [5]. The bag model describes quarks being confined inside a hadron [6]. In high energy collisions, the string model represents the process of particle production in the fragmentation of a stretching string by creating pairs of quark and anti-quark. It worked well [7] for elementary collisions where strings can be formed among the few initial partons and break up to form the soft final state hadrons. However, in relativistic heavy ion collisions (RHIC), there are thousands initial partons. It is intractable to pair partons and have a string for each pair. Even if the strings are formed, they must be modified by the presence of many other color charges. The independent fragmentation approach [7], though valid for high Q^2 partonic processes in the vacuum, cannot explain experimentally observed large p/π ratio in central $Au + Au$ collisions at RHIC [8]. Another possible hadronization mechanism, the deconfined quark (fled out from the bag) recombination model, has been able to reproduce spectra for almost all stable particles for different colliding systems. It provides a natural explanation for the baryon/meson ratio and the nuclear suppression factor observed at RHIC [9].

A natural mechanism for quark confinement is given by the bag model [6]. While the bag model has a few different versions, we shall in this paper keep the essential characteristics of the phenomenology of quark confinement. The gluons are mediate bosons which transfer

the interactions between quarks. Their effect can be replaced by the bag pressure which confines the quarks in a hadron. On the other hand, if all interactions among partons in the bag are neglected, we assume that the partons might be treated as a thermally equilibrated system with a given volume. Then properties of a hadron can be investigated and some characteristic parameters for the hadron can be obtained in the bag model.

The organization of this paper is as follows. In Sec. II, we will discuss some features about a thermally equilibrated QGP system. Then in Sec. III, we get an estimate of the maximum kinetic energy of a confined quark in a spherical cavity of radius R . Combining the discussions in section II and III by assuming its origin from the contribution of gluons, the magnitude of the hadronic bag pressure is discussed in Sec. IV. The last section is for conclusions and discussions.

II. FREE EQUILIBRATED QGP

Let us first consider a thermally equilibrated quark-gluon plasma (QGP) system at first. When its temperature T and volume V are given, the total energy and particle number can be calculated:

$$\begin{aligned} E &= \sum_{i=-N_f}^{N_f} \frac{g_i}{(2\pi\hbar)^3} \int f_i(T) p^0 d\Gamma \\ &= \sum_{i=-N_f}^{N_f} \frac{g_i V}{2\pi^2 \hbar^3} \int f_i(T) p^0 |\mathbf{p}|^2 dp, \end{aligned} \quad (1)$$

$$\begin{aligned} N &= \sum_{i=-N_f}^{N_f} \frac{g_i}{(2\pi\hbar)^3} \int f_i(T) d\Gamma \\ &= \sum_{i=-N_f}^{N_f} \frac{g_i V}{2\pi^2 \hbar^3} \int f_i(T) |\mathbf{p}|^2 dp, \end{aligned} \quad (2)$$

where N_f is the number of flavors and $g_i = N_c N_s$ is the degeneracy number for a parton and equals to the product of quantum numbers of quark's color and spin. f_i is the distribution function which is of Fermi-Dirac for

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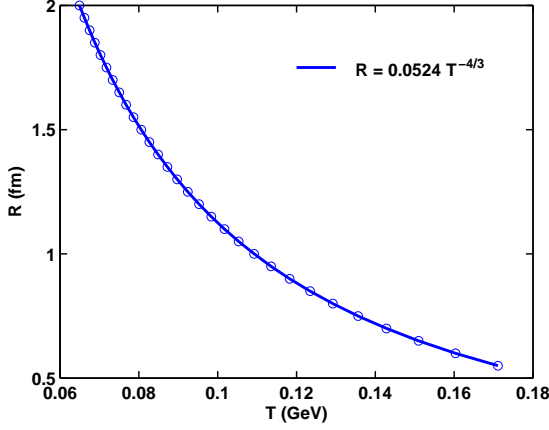


FIG. 1: The relation between the radius of a proton and its temperature when take its mass as the total energy. The numerical simulative formula is given on the legend as well.

quarks and Bose-Einstein for gluons

$$f_i = \frac{1}{1 + e^{(p^0 \mp \mu_q)/T}} \quad \begin{array}{l} \text{'-' for quark,} \\ \text{'+' for anti-quark} \end{array} \quad (3)$$

$$f_i = \frac{1}{e^{p^0/T} - 1} \quad \text{for gluon.} \quad (4)$$

Here μ_q is the quark's chemical potential. For the case when the number density of the quarks is the same as that of the anti-quarks, $\mu_q = 0$.

For a massless quark gas with zero chemical potential $\mu_q = 0$, Eqs.(1,2) can be solved analytically. For example for the case with only two flavors, we get the densities for energy and quark number as ($\hbar = 1$)

$$\begin{aligned} \epsilon = \frac{E}{V} &= \frac{7}{4}(g_q + g_{\bar{q}}) \frac{\pi^2}{30} T^4 + g_g \frac{\pi^2}{30} T^4 \\ &= \frac{37}{30} \pi^2 T^4, \end{aligned} \quad (5)$$

$$\begin{aligned} n = \frac{N}{V} &= \frac{3\zeta(3)}{2\pi^2} (g_q + g_{\bar{q}}) T^3 + \frac{g_g \Gamma(3) \zeta(3)}{2\pi^2} T^3 \\ &= \frac{34 \times 1.202}{\pi^2} T^3. \end{aligned} \quad (6)$$

Now if we treat a proton as a free equilibrated QGP system as use the above results, we can get the relation of the proton radius and temperature. The result is shown in Fig. (1).

From Fig.(1), we can see, the radius of a proton is decreased with its internal temperature. The numerical simulative formula is

$$RT^{4/3} = 0.0524 \quad (7)$$

If the temperature is 100 MeV, the corresponding radius of proton is about 1 fm. If a proton is compressed to have a radius of 0.6 fm, the inner temperature will be about 170 MeV, which is close to the critical temperature, so

that a proton may break and quarks may flee out from the bag.

III. QUARKS CONFINED IN A HADRON BAG

Let's give some discussions about quark wave-function from theory. We assume the quarks are confined in a spherical cavity of radius R, they are free fermions inside it but cannot fly out because all contributions from gluons are attributed to the bag constant. Then the surface of the sphere bag becomes the maximum range that quarks can arrive. On the bag boundary where the current of fermion must be zero. A hadron's wave-function is then product of those for quarks.

The Dirac equation for a free massless fermion in the bag is

$$(i\gamma^\mu \partial_\mu - m)\psi = 0 \quad \text{with} \quad m = 0, \quad (8)$$

where $\partial_\mu = (p^0, \mathbf{p})$. We will in this paper use the Dirac representation

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix},$$

and

$$\gamma^i = \begin{pmatrix} 0 & \sigma^i \\ -\sigma^i & 0 \end{pmatrix},$$

where I is a 2×2 unit matrix and σ^i are the Pauli matrices. We write the four-component wave function for the massless fermion ψ as

$$\psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$

where both ψ_+ and ψ_- are two dimensional spinors. Eq.(8) becomes

$$\begin{pmatrix} p^0 & -\sigma \cdot \mathbf{p} \\ \sigma \cdot \mathbf{p} & -p^0 \end{pmatrix} \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = 0 \quad (9)$$

The lowest energy solution for the above equation is the $s_{1/2}$ state given by[10]

$$\begin{aligned} \psi_+(\mathbf{r}, t) &= \mathcal{N} e^{-ip^0 t} j_0(p^0 r) \chi_+ \\ \psi_-(\mathbf{r}, t) &= \mathcal{N} e^{-ip^0 t} \sigma \cdot \hat{\mathbf{r}} j_1(p^0 r) \chi_-, \end{aligned}$$

where j_l is the spherical Bessel function which can be expressed by an elementary function

$$j_l(x) = (-1)^l x^l \left(\frac{1}{x} \frac{d}{dx} \right)^l \frac{\sin x}{x}, \quad (10)$$

χ_\pm are two-dimensional spinors, and \mathcal{N} is a normalization constant. The confinement of the quarks is equivalent to the requirement that the normal component of the vector current $J_\mu = \bar{\psi} \gamma_\mu \psi$ vanishes at the surface. This

condition is the same as the requirement that the scalar density $\bar{\psi}\psi$ of the quark vanishes at the bag surface $r = R$. This leads to

$$\bar{\psi}\psi|_{r=R} = [j_0(p^0 R)]^2 - \sigma \cdot \hat{\mathbf{r}} \sigma \cdot \hat{\mathbf{r}} [j_1(p^0 R)]^2 = 0$$

or

$$[j_0(p^0 R)]^2 - [j_1(p^0 R)]^2 = 0 \quad (11)$$

From Eq.(10), solution of the above equation is given by

$$p_m^0 R = 2.04, \quad \text{or} \quad p_m^0 = \frac{2.04}{R}. \quad (12)$$

This result means that in order to keep the bag from being broken, the kinetic energy of any quark can not larger than p_m^0 determined by the radius of the bag.

IV. BAG PRESSURE

Take p_m^0 as the upper limit, we can separate the energy of quarks from the total of a proton

$$E_q = \frac{(g_q + g_{\bar{q}})V}{2\pi^2\hbar^3} \int_0^{p_m^0} \frac{p^3 dp}{1 + e^{p/T(R)}}. \quad (13)$$

For simplicity, we have neglected the chemical potential, and treat the quarks as massless. Then $g_q = g_{\bar{q}} = N_c N_s N_f = 3 \times 2 \times 2 = 12$.

The energy from gluon contribution provides the pressure effect directed from the region outside the bag

$$B = \frac{M - E_q}{V}. \quad (14)$$

Here M is the mass of the hadron.

From Eq.(14), we can easily learn that the the bag pressure will change with radius as shown in Fig.(2). We also give the numerical formula

$$B^{1/4} = 0.17R^{-0.65}. \quad (15)$$

The average kinetic energy of each quark can be calculated as

$$\bar{E}_q = \frac{E_q}{N_q} = \frac{\int_0^{p_m^0} \frac{p^3 dp}{1 + e^{p/T(R)}}}{\int_0^{p_m^0} \frac{p^2 dp}{1 + e^{p/T(R)}}}. \quad (16)$$

The energy carried by the valence quarks in a proton is then $3 \times \bar{E}_q$. So that contributions from gluons and sea quarks to the energy is $M - 3\bar{E}_q$. The bag pressure decreases with the radius. This may be used to explain why the resonance particle (which has large radius thus small bag pressure) is usually unstable because their bags are more fragile.

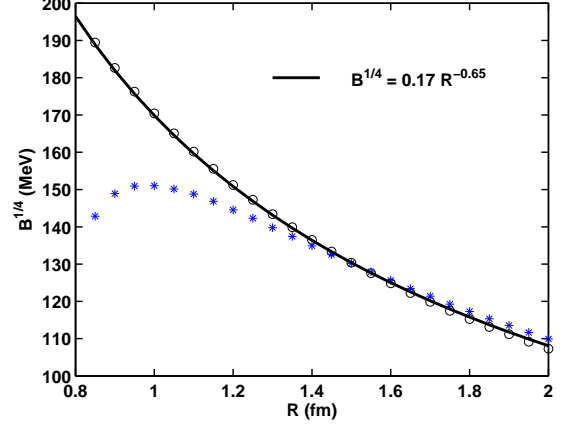


FIG. 2: The bag pressure change with the radius of proton. The open circle is calculated with Eq.(14), and the line is its numerical simulation results, while the star is with the scenario that three quarks are surrounded by gluons.

V. CONCLUSION

In this work we have discussed some feature of a proton from the bag model. Especially, we got the relations between temperature and the radius of the proton when a proton is treated as a non-interacting thermal equilibrium QGP system. The bag pressure comes from the contribution of gluons.

Acknowledgments

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[1] Marc Vanderhaeghen, Nucl. Phys.A 805:210-220,2008.
[2] J. Sowinski, Nucl.Phys.A 790:485-488,2007,
[3] A. Bhattacharya, S. N. Banerjee, B.Chakraborti, S. Banerjee, Nucl. Phys. Proc. Suppl. 142 (2005) 13-15
[4] F. Wilczek, Ann. Rev. Nucl. and Part. Sci. 32, 177(1982)
[5] X. Artru and G. Mennessier, Nucl. Phys. B70, 93(1974),B.Andersson, G. Gustafson and B. Söderberg, Z. Phys. C20,317(1983)

[6] C. D. Detar and J. F. Donoghue, Ann. Rev. Nucl. Part. Sci. 33,235(1983)
[7] Torbjörn Sjöstrand, Leif Lönnblad, Stephen Mrenna, Peter Skands, JHEP 0605(2006) 026
[8] R. C. Hwa and C. B. Yang, J. Phys. G30(2004) S1117-S1120
[9] C. B. YANG, Int. J. Mod. Phys. E 16, No. 10, 3148(2007)
[10] C. Y. Wong, *Introduction to High-Energy Heavy ion Col-*

- lisions*, World Scientific Co., Singapore, 1994.
- [11] Z. G. Tan, S. Terranova and A. Bonasera, Int. J. Mod. Phys. E17 No 8(2008)1577-1589;
- [12] Z. G. Tan, A. Bonasera, C. B. Yang, D. M. Zhou and S. Terranova. Int. J. Mod. Phys. E16, Nos.7&8 (2007) 2269-2275